
Sampling and analytical approaches toward propagating uncertainties in LCA

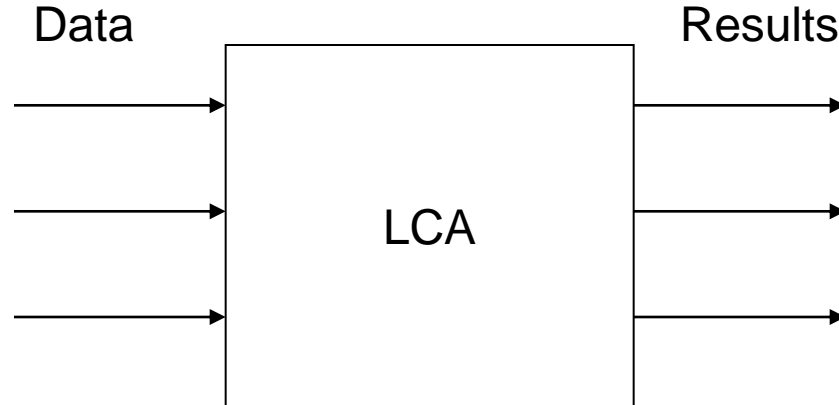
Reinout Heijungs, Olivier Jolliet,
Ralph Rosenbaum, Andreas Citroth,
Thomas McKone, Manfred Lenzen,
Jinglan Hong, Fausto Freire

Contents

- Uncertainty in LCA
- Uncertainty propagation
 - analytical
 - sampling
- Further work

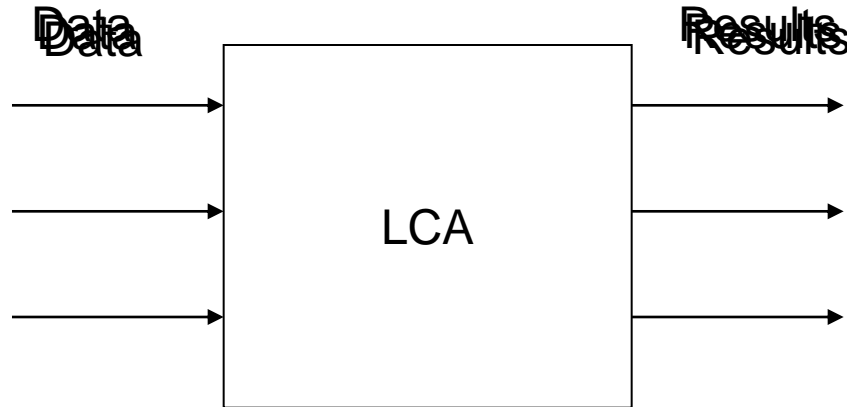
Uncertainty in LCA (1)

■ The LCA model with crisp data



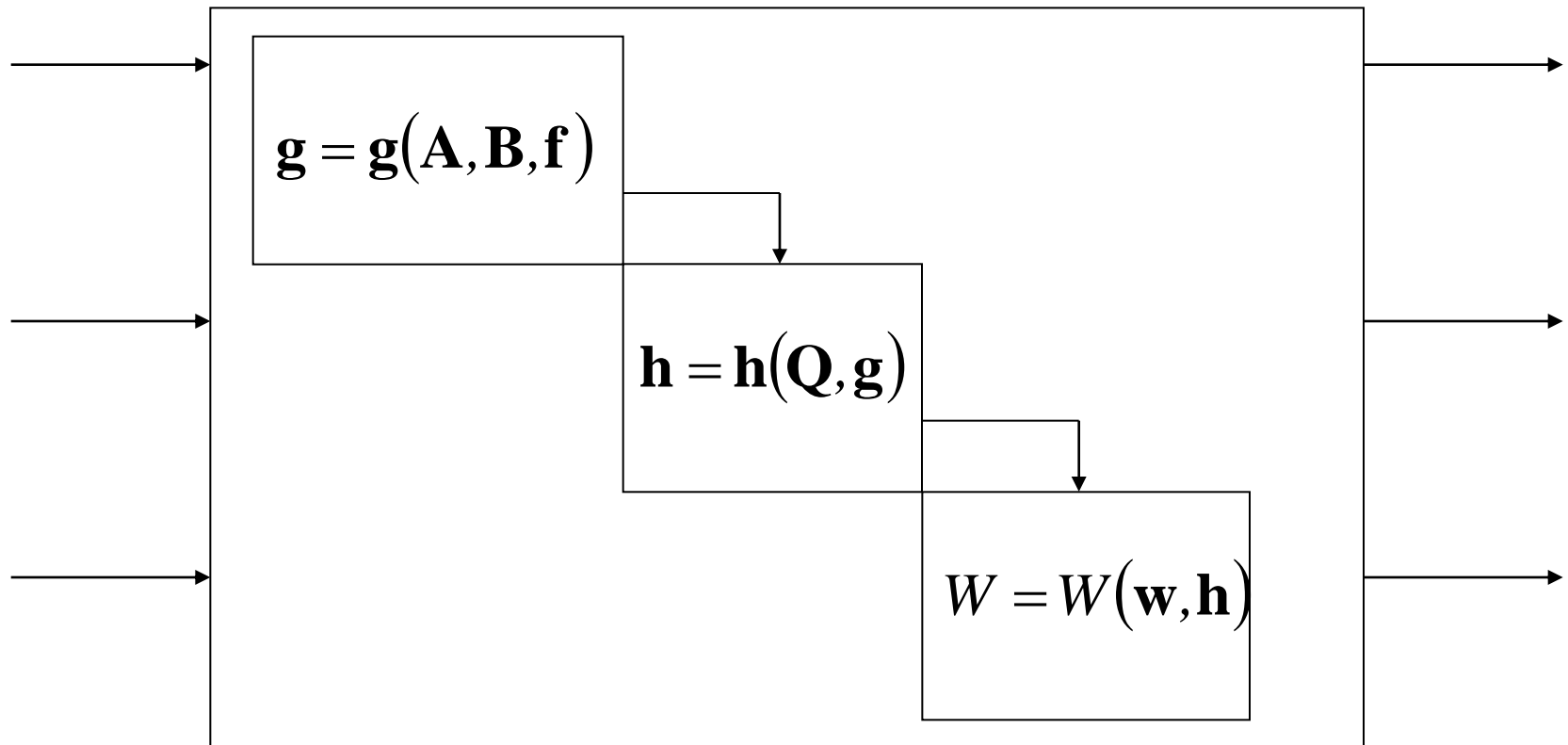
Uncertainty in LCA (2)

■ The LCA model with blurred data



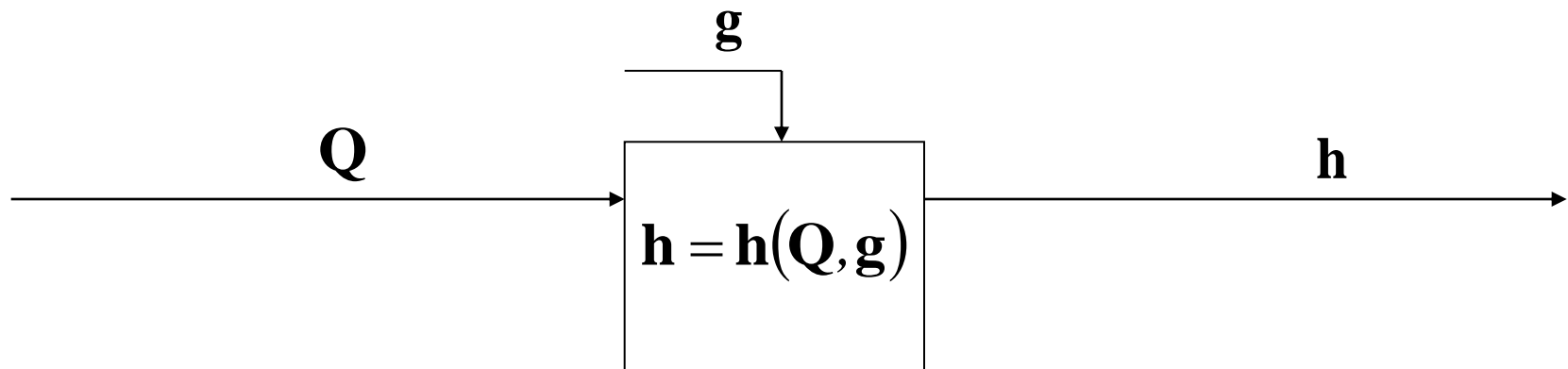
Uncertainty in LCA (3)

- The LCA model, a look inside



Uncertainty in LCA (4)

- The LCIA sub-model, a look inside



Uncertainty in LCA (5)

■ The LCIA sub-model, a look inside

□ typical form of function **h**

$$\mathbf{h} = \mathbf{h}(\mathbf{Q}, \mathbf{g})$$

$$\mathbf{h} = \mathbf{h}(\mathbf{Q}, \mathbf{g}) = \mathbf{Q}\mathbf{g}$$

$$h_{cat} = \sum_{subs} Q_{cat,subs} \times g_{subs}$$

□ example

$$h_{GW} = \sum_{GHG} GWP_{GW,GHG} \times emis_{GHGs}$$

Uncertainty propagation (1)

- Statement of the problem:

- given blurred inventory results \mathbf{g}
- and blurred characterisation factors \mathbf{Q}
- how to find blurred characterisation results \mathbf{h}

- In other words

- how to propagate input uncertainties into output uncertainties

Uncertainty propagation (2)

■ Example

□ $\text{emis}_{\text{CO}_2} = 9 \text{ kg}, 10 \text{ kg}, 11 \text{ kg}, \dots$

□ $\text{emis}_{\text{CH}_4} = 0.9 \text{ kg}, 1 \text{ kg}, 1.1 \text{ kg}, \dots$

□ $\text{GWP}_{\text{CO}_2} = 1$

□ $\text{GWP}_{\text{CH}_4} = 21, 23, 25$

□ $h_{\text{GW}} = ?$

Uncertainty propagation (3)

■ Approaches

□ Sampling

- run the model a number of times to obtain a sample of results

□ Analytical

- run the model only once, but use mathematics to find out information on the degree of blur

Uncertainty propagation - sampling (1)

■ Sampling

□ parameter variation

- run the model for every, or every extreme, set of parameters

□ probabilistic treatment

- run the model for a representative sample of the parameter space

Uncertainty propagation - sampling (2)

■ Example for extreme values

□ minimum

- $\text{emis}_{\text{CO}_2} = 9 \text{ kg}$, $\text{emis}_{\text{CH}_4} = 0.9 \text{ kg}$
- $\text{GWP}_{\text{CO}_2} = 1$, $\text{GWP}_{\text{CH}_4} = 21$
- $h_{\text{GW}} = 27.9 \text{ kg}$

□ maximum

- $\text{emis}_{\text{CO}_2} = 11 \text{ kg}$, $\text{emis}_{\text{CH}_4} = 1.1 \text{ kg}$
- $\text{GWP}_{\text{CO}_2} = 1$, $\text{GWP}_{\text{CH}_4} = 25$
- $h_{\text{GW}} = 38.5 \text{ kg}$

□ range $h_{\text{GW}} = 27.9 - 38.5 \text{ kg}$

Uncertainty propagation - sampling (3)

■ Example for all combinations of values

□ $\text{emis}_{\text{CO}_2} = \{9, 10, 11\}$ kg, $\text{emis}_{\text{CH}_4} = \{0.9, 1, 1.1\}$ kg

□ $\text{GWP}_{\text{CO}_2} = 1$, $\text{GWP}_{\text{CH}_4} = \{21, 23, 25\}$

□ $h_{\text{GW}} = \{27.9, 28.9, 29.7, 29.9, 30, 30.7, 31, 31.5, 31.7, 32, 32, 32.1, 32.5, 33, 33.1, 33.5, 34, 34, 34.1, 34.3, 35, 35.3, 36, 36.3, 36.5, 37.5, 38.5\}$ kg

Uncertainty propagation - sampling (4)

■ Example for probabilistic treatment

□ assuming a distribution

- $\text{emis}_{\text{CO}_2} \sim \text{Gaussian}(\mu = 10 \text{ kg}, \sigma = 1 \text{ kg})$
- $\text{emis}_{\text{CH}_4} \sim \text{Gaussian}(\mu = 1 \text{ kg}, \sigma = 0.1 \text{ kg})$
- $\text{GWP}_{\text{CO}_2} = 1, \text{GWP}_{\text{CH}_4} \sim \text{Gaussian}(\mu = 23 \text{ kg}, \sigma = 2 \text{ kg})$

□ Running Monte Carlo, $N = 1000$

- sample of h_{GW} with mean = 33.1 kg, standard deviation = 2.6 kg

Uncertainty propagation - sampling (5)

	Advantage	Disadvantage
parameter variation - extremes only	2 computations only	don't know which values combine into extremes
parameter variation - all values	straightforward, full spectrum of results	" 10^{23} " computations
probabilistic treatment	representative results	distributions required, quite a few computations

Uncertainty propagation - analytical (1)

■ Analytical

□ perturbation theory

- run the model only once

$$h_{GW} = emis_{CO_2} + GWP_{CH_4} \times emis_{CH_4}$$

- use mathematics to find out information on the degree of blur

$$h'_{GW} = emis'_{CO_2} + GWP_{CH_4} \times emis_{CH_4}$$

$$h'_{GW} = h_{GW} + \Delta emis_{CO_2} + \Delta GWP_{CH_4} \times emis_{CH_4} + GWP_{CH_4} \times \Delta emis_{CH_4}$$

$$h'_{GW} = emis_{CO_2} + GWP'_{CH_4} \times emis_{CH_4}$$

$$\Delta h_{GW} = \Delta emis_{CO_2} + \Delta GWP_{CH_4} \times emis_{CH_4} + GWP_{CH_4} \times \Delta emis_{CH_4}$$

$$h'_{GW} = emis_{CO_2} + GWP_{CH_4} \times emis'_{CH_4}$$

change in \bar{h}_{GW} impact

change in CO_2 emission

change in GWP of CH_4

change in CH_4 emission

Uncertainty propagation - analytical (2)

■ Analytical

□ theory of error propagation

$$\mathbf{h} = \mathbf{h}(\mathbf{Q}, \mathbf{g})$$

$$\text{var}(\mathbf{h}) = \left(\frac{\partial \mathbf{h}}{\partial \mathbf{Q}} \right)^2 \text{var}(\mathbf{Q}) + \left(\frac{\partial \mathbf{h}}{\partial \mathbf{g}} \right)^2 \text{var}(\mathbf{g}) + 2 \left(\frac{\partial \mathbf{h}}{\partial \mathbf{Q}} \right) \left(\frac{\partial \mathbf{h}}{\partial \mathbf{g}} \right) \text{cov}(\mathbf{Q}, \mathbf{g})$$

Uncertainty propagation - analytical (3)

■ Some theoretical issues

- first order linear approximation, small deviations
- covariance often unknown, set to zero
- what are the sensitivity coefficients $\left(\frac{\partial \mathbf{h}}{\partial \mathbf{Q}}\right)$ and $\left(\frac{\partial \mathbf{h}}{\partial \mathbf{g}}\right)$?

Uncertainty propagation - analytical (4)

- Approaches to find out sensitivity coefficients
 - numerical (Hong & Jolliet)
 - matrix derivatives (Heijungs)
 - multiplicative sequential (Ciroth)
- Relative strengths and weaknesses of these approaches still to be found out

Further work

- Further work required
 - when a sampling approach, when an analytical one?
 - if sampling, which?
 - if analytical, which?
- Theoretical and practical investigation
 - biodiesel case