Analytical Uncertainty Propagation in Life Cycle Inventory and Impact Assessment: high-efficiency versus conventional hand dryers

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1. Introduction

- Uncertainty analysis is essential to inform the decision maker on the reliability of the information.

- Typically for an LCA want to know the degree of confidence in the information that impact of scenario A is lower than B or $A/B < 1$.

- Different types of uncertainty (Model, parameter) + variability. Focus is to look at the contribution of parameter uncertainty.
2. Method Monte Carlo and sensitivity

Monte-Carlo

- Advantage to avoid to define output distribution…but
- Rather resource intensive
- Difficult to assess contributions of individual parameters
- Only accounts for parameter uncertainty

Alternative approaches to effectively estimate uncertainty contributions are highly needed.

Combining uncertainty propagation in LCI and LCIA phases is important

Sensitivity

\[
S_i = \frac{\% \Delta \text{Output}_i}{\% \Delta \text{Input}_i} = \frac{\Delta O/O}{\Delta I/I}
\]

1 variable at a time
do not account for inputs uncertainty
Log-normal distributions:

Lognormal:
- $\mu$ is the median
- $GSD^2$ is the Geometric squared standard deviation or coefficient of variation:

$$GSD^2$$

$$\frac{\mu}{GSD^2}$$

$$\mu*GSD^2$$

$$probability \left\{ \frac{\mu}{GSD^2} < X < GSD^2 \cdot \mu \right\} = 0.95$$

if $GSD^2 = 2$, 95% twice lower to twice higher
Method- Taylor series expansion

\[ \text{Geometric standard deviation on output} \]

\[ GSD_O^2 = \exp \left[ \sum_i S_{I_i}^2 (\ln GSD_{I_i}^2)^2 \right]^{1/2} \]

Sensitivity to input parameter \( i \)

Assumptions:
1) Lognormal distribution
2) Independence of all inputs
3) Linear first-order kinetics

MacLeod et al., 2002, More general form by Heijungs et al., 1995
Comparison of 2.5% lower and 95% upper limit: Taylor vs Monte-Carlo: front end panel
Case study: hands dryer

**Function:**
- XLERATOR Dryer: 10s.
- Conventional Dryer: 30s.
- Paper Towels

**Functional Unit:**
- 1 pair dried hands

**Objective:**
Compare the climate change impact of three types of hand dryers

**Comparison:**
- XLERATOR Dryer: transport over car lifetime
- Conventional Dryer
- Paper Towels
Climate change impacts

![Graph showing climate change impacts for different scenarios.](chart.png)
Input data

Square of the geometric standard deviation
(95% confidence interval: between $\mu/SD_{95}$ and $\mu/SD_{95}$)

$$SD_{95} = \exp\sqrt{\ln(U_1)^2+\ln(U_2)^2+\ln(U_3)^2+\ln(U_4)^2+\ln(U_5)^2+\ln(U_6)^2+\ln(U_b)^2}$$

*Ub* Basic uncertainty factor

*U1* Uncertainty factor for reliability,
*U2* Uncertainty factor for completeness,
*U3* Uncertainty factor for temporal correlation,
*U4* Uncertainty factor for la geographic correlation,
*U5* Uncertainty factor for other technological correlation,
*U6* Uncertainty factor for sample size,

Example Aluminum:

$$SD_{95} = \exp\sqrt{\ln(1,00)^2+\ln(1,02)^2+\ln(1,00)^2+\ln(1,02)^2+\ln(1,00)^2+\ln(1,20)^2+\ln(1,05)^2} = 1.21$$
Comparison of single scenarios
Based on these distributions, one might think that the probability that scenario A is higher than scenario B is the area of intersection
Distribution: log-normal - overlapping

- HOWEVER, the two scenarios are always dependent → depend on the same parameters.
- Therefore when one set of parameters \((p_1, p_2, p_3)\) yields a high result in scenario A, it is likely to also yield a high result in scenario B.
- Difference is generally more robust in LCA → Run A and B in parallel and determine \(P(A-B>0)\) or \(P(A/B>1)\)
Method - Taylor series expansion scenario comparison A/B

\[
\left( \frac{\ln GSD_A}{B} \right)^2 = \sum_{i} S_{A_i}^2 \left( \ln GSD_i \right)^2 + \sum_{j=i+1}^{m} S_{B_j}^2 \left( \ln GSD_j \right)^2 + \sum_{k=m+1}^{n} \left( S_{A_k} - S_{B_k} \right)^2 \left( \ln GSD_k \right)^2
\]

- Independent parameters for scenarios A and B
- Sum of all parameters
- Common parameters to A and B
  - Take the difference in sensitivity

\[
P \left( \frac{A}{B} < 1 \right) = \frac{1}{2} + \frac{1}{2} \text{erf} \left[ \frac{-\xi_A}{\ln GSD_A \sqrt{2}} \right]
\]
P (A/B<1)

XLERATOR / Standard Dryer

XLERATOR / Paper Towels (0%)

XLERATOR / Paper Towels (100%)

Standard Dryer / Paper Towels (0%)

Standard Dryer / Paper Towels (100%)

Paper Towels (0%) / Paper Towels (100%)
Main parameters contributing to uncertainty

- Xlerator/Standard
- Xlerator/Paper
- Xlerator/100% recycled
- Standard/Paper
- Standard/Recycled
- Paper/Recycled

- Other
- Electronic component
- Paper disposal
- Sulphate pulp
- Transport lorry
- Electricity US
Test of log normality: the only hypothesis is that the output is lognormal.
5. Conclusions

- Demonstrates the feasibility of this method and illustrates its simultaneous and consistent application to both inventory and impact assessment.

- This simple and reliable approach can easily show the contribution of each process.

- This detailed and modular approach coupled with a LCA is a very relevant and efficient way to get an accurate overall LCA uncertainty.

- The fairly simple procedure very strongly reduce calculation time. Because the approximate method relies on representative GSD2 and therefore needing little calculation resources was presented.
2. Methodology-Approach method

- Determine the output sensitivity to each input parameter

- Assesses the overall coefficient of variation on the final result as a function of the coefficient of variation of each individual input.

- The advantage of this procedure is to explicitly provide the contribution from each parameter as well as very strongly reduce calculation time.

- Compare results with Monte-Carlo analysis. Probability distributions obtained with this approach are compared to classical Monte Carlo distributions for test scenarios.
Case study Car front end panel

Car front-end panel

Function:
transport over car lifetime

Functional Unit:
1 front-end panel of equivalent rigidity for a 200'000 km service.

Objective:
Compare the climate change impact of a **steel** versus an **aluminium** front end panel
1. Introduction

2. Method

3. Single scenario

4. Comparing scenario

5. Conclusions

Process tree – steel (263 kg CO₂equ/FU)

15.5 kg Steel
GSD² = 1.1

39 kWh
GSD² = 1.1

99 MJ
GSD² = 1.1

80 l gasoline
GSD² = 1.03

CO₂ emissions
2.4 kg CO₂/kg gas
GSD² = 1.1
Process tree – aluminium (173 kg CO$_2$ eq/FU)

- 5.9 kg Alu, GSD$^2$=1.1
- 16 kWh, GSD$^2$=1.1
- 77 MJ, GSD$^2$=1.1
- CO$_2$ emissions: 2.4 kg CO$_2$/kg gas, GSD$^2$=1.1
- 30 l gasoline, GSD$^2$=1.03
4. Results - For single scenario

- **Monte carlo**
  - Steel: $GSD^2 = 1.11 - 1.12$
  - Alu.: $GSD^2 = 1.08 - 1.10$

- **Taylor series**
  - Steel: $GSD^2 = 1.09$
  - Alu.: $GSD^2 = 1.10$
This Taylor extension serie is appropriate to easily determine the contribution of each process.
Relationship between Taylor serie - Monte Carlo for GSD$^2$ in single scenario

GSD$^2$ calculated by using Taylor serie consistent with Monte carlo
1. Introduction
2. Method
3. Single scenario
4. Comparing scenario
5. Conclusions

**Process tree – steel** (263 kg CO\textsubscript{2} equ/FU)

- **15.5 kg Steel**
  - GSD\textsuperscript{2} = 1.1

- **39 kWh**
  - GSD\textsuperscript{2} = 1.1

- **99 MJ**
  - GSD\textsuperscript{2} = 1.1

- **80 l gasoline**
  - GSD\textsuperscript{2} = 1.03 $\rightarrow$ 1.77

Independent CO\textsubscript{2} emissions: 2.4 kg CO\textsubscript{2}/kg gas

Dependent CO\textsubscript{2} emissions: GSD\textsuperscript{2} = 1.1 $\rightarrow$ 2.0
Equal uncertainty on equal scenario, independent vs dependent

**A-B>0?**

**Varying the same parameter (dependent)**

Probability

A-B<0 = 0.2%

**Varying independent parameters**

Probability

A-B<0 = 6%

**Scenario**

Steel-Alu. Crosses Zero
For comparing two scenarios

\[ \ln \frac{GSD_A^2}{B} = \ln GSD_A^2 + \ln GSD_B^2 + 2 \text{Cov} (\ln Y_B, \ln Y_B) \]

\[ \frac{GSD_A^2}{GSD_B^2} = \frac{GSD_A^2}{GSD_B^2} \leq GSD_A^2 GSD_B^2 \]

- Fully positively correlated
- Independent
- Fully negatively correlated

\[ GSD_{Steel}^2 \text{Alu.} = 0.99 - 1.14 \]
Results - For comparison two scenarios: Steel / Alu.

Fig. 3 Comparison of Monte-Carlo and Taylor series for steel-aluminum scenario

Taylor series
- Fully positively correlated
- Fully negatively correlated
- Independent

Since, most cases in LCA is positively correlated

\[
\frac{GSD_{\text{Steel}}^2}{GSD_{\text{Alu}}^2} = 0.99 - 1.14
\]